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LETTER TO THE EDITOR

Orthogonality catastrophe in a composite fermion liquid

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Abstract. We discuss the emergence of an orthogonality catastrophe in the response of a composite fermion liquid as the filling factor ν approaches $1/2m$, where $m = 1, 2, 3 \ldots$. A tunnelling experiment is proposed in which dramatic changes in the I–V characteristic should be observable as ν is varied. Explicit I–V characteristics calculated within the so-called modified random phase approximation, are provided for $\nu = \frac{1}{3} \to \frac{1}{2}$.

Composite fermion theory [1–4] has been remarkably successful in explaining the fractional quantum Hall effect in terms of the integer quantum Hall effect of a composite fermion metal [5, 6]. Within this theory, the ground state of a two-dimensional electron gas (2DEG) at even-denominator filling factors $\nu = 1/2m,\ m = 1, 2, 3 \ldots$, is a compressible Fermi liquid containing composite fermions which experience zero effective magnetic field $B^\star$ [2–4]. Various recent experiments on systems near $\nu = \frac{1}{2}$ have indeed been interpreted in terms of a Fermi sea of composite fermions [5, 6]. Further confirmation of the details of the theory as $\nu \to 1/2m$ is still however desirable.

In this letter, we propose an experiment to probe the emergence of the Fermi surface in a composite fermion liquid as $\nu \to 1/2m$. Mono-energetic electrons are allowed to tunnel into a quantum dot placed close to the 2DEG. We find the signature of the presence of a Fermi surface at $\nu = 1/2m$ to be a dramatic 'orthogonality catastrophe' in the tunnelling current (I) as a function of the gate voltage (V) of the dot. For $\nu \neq 1/2m$, strong oscillatory signatures in the I–V characteristic are predicted—these signatures are ν-dependent and can be used to deduce the composite fermion effective mass. In addition, the spectrum for odd-denominator fractions will yield valuable information about the 'gapfull' excitations of these states.

Our proposed experiment is analogous to inverse photoemission spectroscopy (IPS) [7, 8] experiments on ordinary metals, with the atomic core level replaced by a quantum dot. In an IPS experiment, a free electron falls into a hole in an atomic core state thereby emitting a photon of energy ω_0. The intensity of the emitted photon is identically zero at the threshold energy ω_0. This is a consequence of the so-called 'orthogonality catastrophe' (OC); the transition involved is forbidden because the initial and final states are orthogonal [7–11].

Figure 1 provides a schematic illustration of our proposed experiment. Quantum dot A acts as an electron monochromator, because only source electrons with energy ϵ can resonantly tunnel to A. An electron in A will resonantly tunnel to quantum dot B only if there are states available with energy ϵ. In the absence of the 2DEG the density of states of B is a δ-function at energy ϵ_0, and tunnelling occurs only when $\epsilon = \epsilon_0$. However, the presence of the 2DEG means that the density of states is asymmetrically broadened to higher

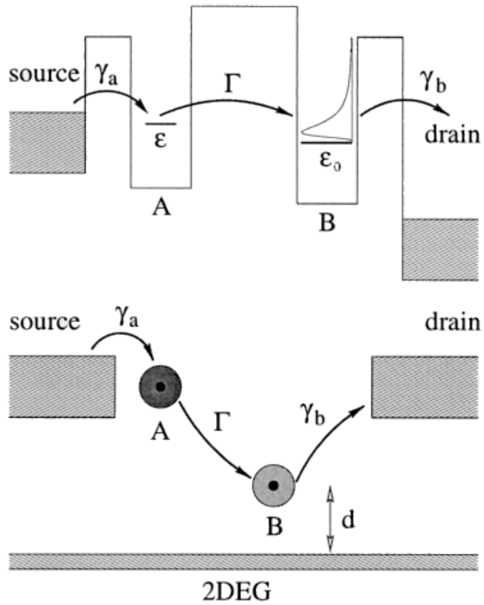

Figure 1. Schematic diagram of the junction. A gate voltage V is applied to dot B to alter the energy of dot B with respect to A. The source-drain voltage is kept fixed. The plane of the 2DEG is perpendicular to the page.

energies due to the neutral excitations in the 2DEG induced by the filled dot—this implies that tunnelling can occur if $\epsilon > \epsilon_0$. The electron can then tunnel out into the drain lead to be measured as a current, determined by the tunnelling rates γ_a, Γ, and γ_b. The current I is measured as function of the gate voltage V controlling the difference between the energy ϵ of the injected electron and the energy ϵ_0 of dot B. This resonant tunnelling is similar to IPS with a zero energy photon, and the analogue of the IPS spectrum is the tunnelling I–V characteristic; the threshold $V = V_0$ in this case is such that $\epsilon_0 = \epsilon$. Generally, $\epsilon - \epsilon_0 = e(V - V_0)$, hence the creation of excitations implies that the spectrum is non-zero for $V > V_0$.

Once an electron has tunnelled into dot B it must reside there for a time greater than the response time of the 2DEG. This implies that the electron tunnels out with rate γ_b less than the desired resolution, which is typically the composite fermion Landau-level spacing for a filling factor close to $\nu = \frac{1}{2}$. The other two rates Γ and γ_a are determined by the following simple argument. The average current can be written as

$$I = \frac{e}{T}$$

where T is the total time taken to tunnel from source to drain. In terms of the tunnelling rates we have for sequential tunnelling

$$T \sim \frac{1}{\gamma_a} + \frac{1}{\Gamma} + \frac{1}{\gamma_b}.$$

In order for the tunnelling current to reflect only the density of states of dot B, we choose the conditions

$$\gamma_a \sim \gamma_b \equiv \gamma \qquad \Gamma \ll \gamma \qquad \gamma \leqslant \omega_c^\star \tag{1}$$

in which case the current is given by [9]

$$I = e\Gamma = e\Gamma_0 \gamma \,\mathrm{Re} \int_0^\infty \mathrm{d}t \exp\left[\mathrm{i}\frac{e(V - V_0)t}{\hbar} - F(t) - \gamma t \right]. \tag{2}$$

The time-integral in equation (2) gives the convolution of the density of states of dot B with a Lorentzian of width γ representing the broadening due to tunnelling to the drain. In the absence of the 2DEG, the function $F(t) = 0$, and $I(V_0) = e\Gamma_0$. The bare tunnelling rate Γ_0 is determined solely by the width and height of the barrier between dots A and B. A suitable value consistent with equation (1) is $\hbar\Gamma_0 \approx 6\ \mu$eV. The density of states of the electron, once it has tunnelled to dot B, depends on the excitation spectrum of the 2DEG through the function

$$F(t) = \int_0^\infty \mathrm{d}\omega \frac{(1 - \mathrm{e}^{-\mathrm{i}\omega t})}{\omega^2} \rho(\omega)$$

where

$$\rho(\omega) = \frac{1}{\hbar} \sum_{\boldsymbol{q}} |V(\boldsymbol{q})|^2 S(\boldsymbol{q}, \omega)$$

is the density of single pair-excitations of the 2DEG due to the sudden appearance of an electron in B. $V(\boldsymbol{q})$ is the potential experienced by the 2DEG due to the electron in B [7]. The dynamic structure factor $S(\boldsymbol{q}, \omega)$ [12] contains information about the excitation spectrum of the 2DEG, and is calculated using the Chern–Simons theory of composite fermions within the modified random phase approximation (MRPA) [4]. We note that in approximating $F(t)$ as above, we are treating the excitations of momentum $\boldsymbol{q}$ and energy ω as independent bosons, which is standard in the theory of IPS in ordinary metals. The theory includes all such bosonic excitations exactly to all orders in perturbation theory [7].

The MRPA resolves the conflict of requiring both renormalization of the composite fermion mass and satisfaction of Kohn's theorem [2–4, 12]. In the limit where the electron cyclotron energy is large compared with the Coulomb energy, the composite fermion mass is expected to scale as the square-root of the magnetic field. Using the RPA equations with this renormalized mass leads to satisfying neither Kohn's theorem nor the f-sum rule; the MRPA repairs this within Fermi-liquid theory by adding a Landau interaction term. In our calculations we use a renormalized composite fermion effective mass which scales as the square-root of the magnetic field such that

$$m^\star_{\mathrm{CF}} = \frac{4\pi\epsilon_0\epsilon_{\mathrm{r}}\hbar^2}{0.3e^2 l_{\mathrm{c}}} \tag{3}$$

where the magnetic length is l_{c} and the dielectric constant $\epsilon_{\mathrm{r}} = 13$ [2–4].

There are three essential features to be incorporated in the experimental design (see figure 1). First, the probe dot B must be separated from the plane of the 2DEG by a barrier which is sufficiently high to prevent tunnelling between B and the 2DEG. However, B must be as close to the 2DEG as possible because the Fourier transform of the potential experienced by the 2DEG due to an electron at B decays exponentially with the separation d. Second, the source and drain leads and dot A must be far enough away from the 2DEG so that the only potential experienced by the 2DEG comes from the electron at B. Third, the levels in the dots must be well spaced so that only one level contributes to the tunnelling; two-electron tunnelling will be suppressed because of Coulomb blockade. Our calculations are therefore based on a 2DEG with 10^{15} electrons m^{-2} placed $d = 50$ Å away from a dot with a confinement length of 50 Å.

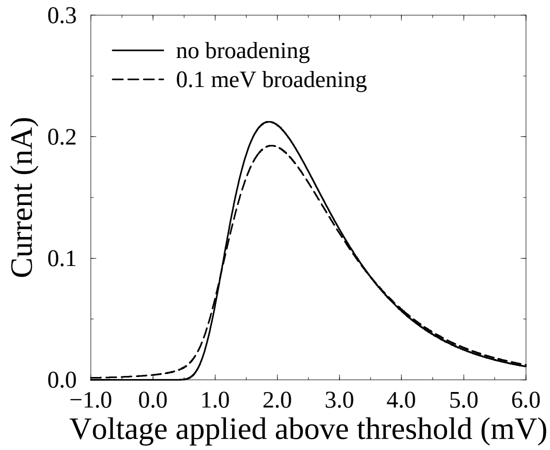

Figure 2. Current at $\nu = \frac{1}{2}$ as a function of the voltage applied above threshold, i.e. $V - V_0$. Solid curve: spectrum in limit of perfect resolution, i.e. no instrumental broadening. Dashed curve: current with a broadening of 0.1 meV.

Figure 2 shows the I–V characteristic calculated for the compressible state with filling factor $\nu = \frac{1}{2}$. The solid line does not include any instrumental broadening thereby emphasizing the suppression at threshold. The dashed curve shows the current with a Lorentzian broadening of width $\gamma = 0.1$ meV; here there is a small current at and below the threshold. The peaked shape is completely different from the power-law which arises for an ordinary metal [7–11]. The OC is clear, because the threshold current is negligible—the low energy excited states have a very small overlap with the initial state and are strongly suppressed, in complete contrast to those in an ordinary metal. The suppression is a consequence of the diffusive mode which dominates the dynamic structure factor at $\nu = \frac{1}{2}$ at low energies and momenta [2]. This mode arises from the scattering of the composite fermions by their attached flux tubes. The sudden appearance of the perturbing potential causes density fluctuations in the 2DEG, which induce modulations in both the scalar and vector potentials resulting from the diagonal and off-diagonal terms in the Chern–Simons interaction; these potentials further scatter the composite fermions thereby creating more density fluctuations. The result is a diffusive mode $\mathrm{i}\omega = \eta \boldsymbol{q}^2$, corresponding to a screening charge density

$$\rho(\boldsymbol{q}, t) \propto 1 - \exp\left(-\frac{t}{\tau}\right)$$

where $1/\tau = \eta \boldsymbol{q}^2$. Thus whereas the characteristic response time for electrons to screen the charge is very short, the response time for the composite fermions is found to diverge in the long-wavelength limit. This is a consequence of there being more low-energy excitations in the composite fermion gas than in an electron gas.

The power-law divergence in ordinary metals arises from a density of pair excitations $\rho(\omega)$ which goes linearly with ω for energies small compared with the Fermi energy [7]. This is not true for the composite fermion metal, where we find $\rho(\omega)$ varies roughly as $\sqrt{\omega}$. One can derive this result analytically using the single-mode approximation employed by He *et al* [13] in their study of tunnelling between two 2DEGs as a function of the bias voltage. It should be noted that our work differs in two ways from that of reference [13]. First, and most importantly, their problem involves the *charged* excitations of the 2DEG because their tunnelling process removes an electron from one 2DEG and places it in the other. They treat the added (removed) electron as an infinitely massive foreign particle inserted into the N-electron system, thereby reducing the problem to that of the x-ray edge problem. Our experiment is, by design, analogous to IPS without further need for approximations because the tunnelling electron is both distinguishable from the 2DEG and localized. Our experiment therefore strictly probes the *neutral* excitations of the 2DEG. Second, our work differs in the level of approximation employed for the dynamic structure factor and the subsequent determination of the density of pair excitations. We use the full MRPA dynamic structure factor, which allows us to calculate the I–V characteristic at both odd- and even-denominator filling factors. By contrast, the single-mode approximation used in reference [13] is valid only at even-denominator filling factors. Applying the single-mode approximation to our experiment (with $d = 0$) we predict a current of the form

$$I \propto \frac{1}{\sqrt{(V - V_0)^3}} \exp\left(\frac{\alpha}{V_0 - V}\right) \qquad V > V_0$$

which is in qualitative agreement with the current calculated numerically.

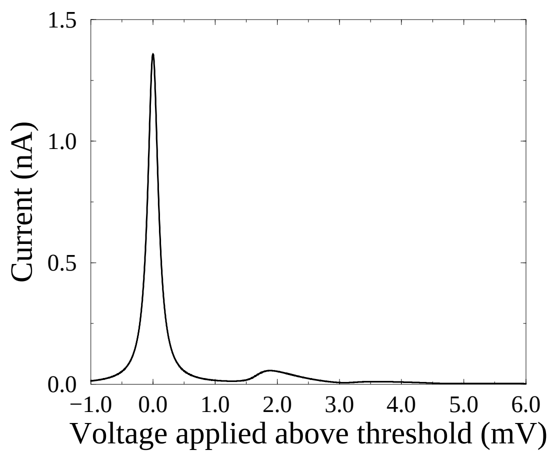

Figure 3. Current calculated for $\nu = \frac{1}{3}$ as a function of the voltage applied above threshold, i.e. $V - V_0$. Finite broadening as in figure 2.

Figures 3, 4 and 5 show the I–V characteristic for the 2DEG at three odd-denominator filling factors originating from the $\nu = \frac{1}{2}$ state. A state at $\nu = p/(2mp + 1)$ can be described in terms of composite fermions at effective filling factor $\nu^\star = p$ interacting with flux tubes carrying $2m$ magnetic flux quanta. We have considered the cases where $m = 1$ and $p = 1$, 3, and 7, which correspond to the fractions $\nu = \frac{1}{3}$, $\frac{3}{7}$ and $\frac{7}{15}$ respectively. The I–V characteristic at $\nu = p/(2mp+1)$ differs in two important respects from that at $\nu = \frac{1}{2}$. First, there is now no orthogonality catastrophe, because there is a gap to excitations. While in the absence of the 2DEG the threshold current is $e\Gamma_0$, the presence of the 2DEG reduces the threshold current by the Debye–Waller factor $\exp(-L)$, where

$$L = \int_0^\infty \mathrm{d}\omega \frac{\rho(\omega)}{\omega^2}.$$

This current decreases as the effective magnetic field is reduced because L increases, signifying the reduction in the overlap of the initial state and final ground state; in the limit of zero effective field $L \to \infty$, implying that the overlap tends to zero yielding the OC.

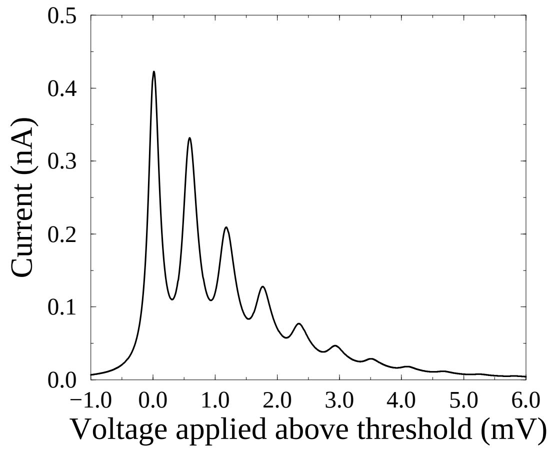

Figure 4. Current at $\nu = \frac{3}{7}$ as a function of $V - V_0$. Finite broadening as in figure 2.

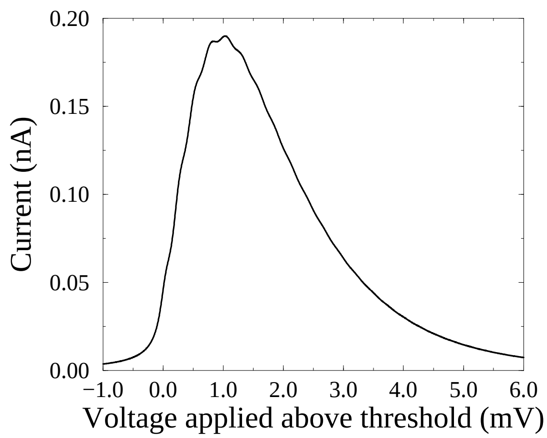

Figure 5. Current at $\nu = \frac{7}{15}$ as a function of $V - V_0$. Finite broadening as in figure 2.

Second, the I–V spectrum develops oscillations superimposed on an envelope similar in shape to the I–V curve for $\nu = \frac{1}{2}$. The period of these oscillations is given by the composite fermion cyclotron frequency $\omega_c^\star = eB^\star/m_{CF}^\star$, where $B^\star = n_e h/ep$. A measurement of this period can thus provide an experimental test of the validity of equation (3).

At high effective fields $B^\star$, such as $\nu^\star = 1$ ($\nu = \frac{1}{3}$), the oscillations dominate the spectrum and the threshold current is large because L is small. As $B^\star$ is reduced L increases, thus the threshold current decreases and the envelope function becomes more distinct. As expected the $B^\star \to 0$ limit resembles the $B^\star = 0$ calculation giving further confidence in our results. The transition from high to low effective field displayed in figures 2–5 is quite dramatic—the peak at threshold changes to a complete suppression. The shape of $\rho(\omega)$ away from $\nu = 1/2m$ reflects the gapfull nature of the excitations, and contains peaks separated by $\omega_c^\star$. When $B^\star$ is small, $\rho(\omega)$ has an envelope which goes roughly as $\sqrt{\omega}$ for low energies and hence tends towards the $B^\star = 0$ (i.e. $\nu = 1/2m$) shape.

We now compare the response of an ordinary metal in a weak magnetic field with the composite fermion system described above [7, 10, 14]. The I–V spectrum for ordinary electrons in zero magnetic field diverges as a power-law, being identically zero at threshold. On applying a weak magnetic field the spectrum develops oscillations with a period equal to the cyclotron frequency but with the divergent shape retained as an envelope. Since there is now a gap to excitations, there is no OC and $I(V_0) = e\Gamma_0 \exp(-L)$. The effect of applying stronger fields is to transfer weight from the higher energy peaks to the threshold peak. Therefore the electron spectrum is always largest around threshold. This is in sharp contrast to the present case of composite fermions where, as shown above, the near-threshold behaviour changes from being completely suppressed in zero effective field (e.g. $\nu = \frac{1}{2}$), to being highly peaked at large effective fields (e.g. $\nu^\star = 1$, and hence $\nu = \frac{1}{3}$).

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